Back paper Examination

Physics III,
B. Math.,
September - December 2020.
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Duration: 2 hours.
Total points: 45.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. (a) A sphere of radius R carries a charge density $\rho(r) = kr$ where k is a constant. There is no charge outside the sphere . Find the electrostatic energy of this configuration. [3]

   (b) A very long cylindrical object consists of an inner solid cylinder of radius a, and it carries a uniform charge density ρ. Cylindrically coaxial to the inner cylinder, there is a thin cylinder of radius b which carries a equal but opposite total charge as the inner cylinder, which is distributed uniformly over its surface.

      i. Find the electric field everywhere. [4]
      ii. Calculate the electric potential everywhere, taking $\Phi = 0$ on the outer cylinder. [4]
      iii. Calculate the total electrostatic energy per unit length of the object in this configuration. [4]

2. (a) A point dipole $\vec{p}$ is embedded of a center of a sphere of radius R. The sphere is filled with a linear dielectric material of dielectric constant ϵ_r. Find the electric potential

      i. inside the sphere [4].
      ii. outside the sphere [3].

   (b) There are two concentric conducting spheres of radii a and b $(a < b)$ such that the inner sphere carries a charge Q and the outer sphere carries a charge $-Q$. The empty space between the two spheres can be imagined to be divided in two equal hemispherical parts: one is filled with a dielectric material with dielectric material with dielectric constant $\epsilon_r(> 1)$, while the other is just free space.

i. The electric field in the space between the two spheres. [3]

ii. The induced surface charge on the inner sphere. [2]

iii. The polarization charge density induced on the dielectric surface at $r = a$. *Note:* The polarization charge will be induced on the dielectric surface, not the conducting surface of the sphere at $r = a$. [3]

3. (a) Imagine that there is a finite volume $\mathbb{V}$ and a surface $\partial\mathbb{V}$ enclosing it. The volume current density $\vec{J}$ is specified throughout $\mathbb{V}$ while **either** the vector potential $\vec{A}$ **or** the magnetic field $\vec{B}$ is specified throughout $\partial\mathbb{V}$. Prove that the magnetic field is uniquely determined throughout $\mathbb{V}$ with the given information. [6]

(b) A long coaxial cable consists of two coaxial cylindrical tubes, which is separated by a linear insulating material of magnetic susceptibility χ_m. The inner cylinder carries a uniform current $+I$, distributed evenly over the surface while the outer cylinder carries a uniform current $-I$, again uniformly distributed over the surface.

i. Find the magnetic field in the region between the two cylinders. [4]

ii. Find the magnetization $\vec{M}$ and the bound current(s) in the same region as well as on both surfaces, if any surface current exists [5].